Turbulent Circulation in Bubble Columns

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Introduction

Circulation is encountered in a wide range of multiphase engineering process equipment, but no theoretically sound method exists for predicting the circulation velocities except in very viscous flow systems. This paper presents a new approach to modeling the circulation in turbulent gas-liquid bubble columns, where gas is bubbled through a distributor into a volume of liquid to achieve interphase mass transfer. In the most common type of circulation, called gulfstreaming, liquid carrying a high concentration of the gas bubbles rises upward near the center of the vessel. Liquid carrying fewer bubbles flows back downward near the walls of the vessel. The central gulfstream serves to carry gas through the column more rapidly than if the gas rose as ideal bubbles evenly distributed across the column diameter (Wallis, 1969), and alters the efficiency of the column as a reactor.

Several researchers have accounted for circulation by modeling the overall holdup of gas in the column (Lockett and Kirkpatrick, 1975; Shah et al., 1982; Deckwer et al., 1974), but this approach is not general and cannot be used to infer the actual fluid velocity in the vessel. Velocity distributions above the entrance zone in gas-liquid systems have been measured or proposed by many different workers for a variety of column diameters (Rietema and Ottengraf, 1970; Hills, 1974; Rietema, 1982; Freedman and Davidson, 1969; Lamont, 1958; Steinemann and Buchholz, 1984) but little theoretical analysis is given in the literature. Rietema and Ottengraf presented an analysis for the circulation in a viscous liquid that is not applicable to a turbulent system. Clark (1984) has presented a circulation analysis for Pachuca tanks, but this is valid only for tanks with full-height draft tubes. Hills (1974) examined air-water flow in a column and offered a model that used a force balance on an annular ring in the column; however, the analysis does not model the momentum dispersion very accurately. A more detailed analysis along these lines has been presented by Miyauchi et al. (1981), but it also involves simplifying assumptions about the relationship between shear stress and velocity that may not hold true in real column circulation.

New Analysis

We will develop a model for the distribution of axial velocity at half the height (where radial velocity components may be neglected) of a sufficiently tall bubble column with fully developed turbulent flow. For this modeling we require an axisymmetric distribution of gas void fraction as a function of radius, $\epsilon(r)$, where r is radial distance from the column centerline. For most flow situations this has been shown to assume a shape that is closely fitted by a parabolic or radial power law equation, such as that due to Bankoff (1960).

$$\epsilon(r) = \epsilon_c[1 - (r/R)]^{1/p} \tag{1}$$

where ϵ_c is the void fraction at the column center and R is the column radius; p is a value, typically 7, describing the power law curve.

More recently, experimental and theoretical work in gas-liquid pipe flow has demonstrated that void profiles may differ significantly from the parabolic shape at low gas superficial velocities (Drew and Lahey, 1981, 1982; Beyerlein et al., 1985; Clark and Flemmer, 1985a; Herringe and Davis, 1978), so that other relationships may be needed in place of Eq. 1. Nevertheless, the theory presented below is general and applicable to all void distributions and can be applied equally well to parabolic and to saddle-shaped distributions.

The radial distribution of time-averaged density, $\rho(r)$ is found from the void distribution using the equation

$$\rho(r) = \rho_L[1 - \epsilon(r)] + \rho_G\epsilon(r) \tag{2}$$

where ρ_L and ρ_G are liquid and gas densities and where the second term on the righthand side is often neglected. The distribution of axial shear stress across the diameter is readily predicted using a force balance equation as derived by Levy (1960).

$$T(r) = T_w\{1 + g[\bar{\rho} - \rho_i(r)]/(2T_wR)\}(r/R) \quad (3)$$

where T_w is the wall shear stress, $\bar{\rho}$ is the average density across the whole vessel cross section, and $\rho_i(r)$ is the average density within a central cylinder of radius r:

$$\rho_i(r') = (1/\pi r'^2) \int_0^{r'} 2\pi\rho(r)r\,dr \quad (4)$$

We do not require knowledge of T_w *a priori*, as is demonstrated below.

From mixing length theory (Schlichting, 1968), the shear stress is given by

$$T(r) = -\rho(r)L(r)|dU(r)/dr|(dU(r)/dr) \quad (5)$$

where the absolute value signs are inserted to maintain the sign (direction) of the shear, and where L is the mixing length and U is the liquid velocity. With reasonable assumptions, this can be extended to the case of a two-phase flow (Clark and Flemmer, 1985b,c). Density $\rho(r)$ is taken as the mixture density at each value of radius. This is equivalent to assuming that momentum transfer takes place only in the liquid phase and that the liquid phase has constant density ρ_L. Let the velocity U be the actual (not superficial) liquid velocity at a point in the vessel, in which case by equating the shear stress given by Eqs. 3 and 5 it is possible to predict the first derivative of this velocity with respect to radius.

$$[dU(r)/dr]|dU(r)/dr| = [1/L(r)][1/\rho(r)] \cdot T_w\{1 + g[\bar{\rho} - \rho_i(r)]/(2T_wR)\}(r/R) \quad (6)$$

Now, by selecting a suitable boundary condition (usually by analogy with the universal velocity profile) where the velocity $U(r)$ can be taken as zero at some finite distance from the wall, y_w, it is possible to determine the liquid velocity at any radius r' by integrating the square root of the righthand side of Eq. 6 and retaining the appropriate sign for $dU(r)/dy$ in relation to Eq. 5.

$$U(r') = \int_{R-y_w}^{r'} ([1/L(r)]^2[1/\rho(r)]T_w\{1 + g[\bar{\rho} - \rho_i(r)]/ \cdot (2T_wR)\}(r/R)^{0.5}\,dr \quad (7)$$

$$U(R - y_w) = 0$$

A reasonable value for y_w that can be offered for a smooth-walled vessel is

$$y_w = 0.111\nu/(|T_w|/\rho_L)^{0.5} \quad (8)$$

where ν is the kinematic viscosity $\nu = \mu/\rho_L$. A more rigorous boundary condition would involve the standard technique of assigning a value to the velocity at the boundary layer thickness, but the approach offered above is sufficiently accurate, as borne out by agreement with data below. The reader is referred to Hinze (1975) and Schlichting (1968) for boundary conditions near rough walls.

Mixing length, $L(r)$, is assumed to be the same as that for a single-phase flow, given by the equation of Nikuradse as presented by Schlichting (1968).

$$L(r)/R = 0.14 - 0.08(r/R)^2 - 0.06(r/R)^4 \quad (9)$$

Although determined for nonreversing pipe flow, Eq. 9 proved accurate in the modeling, as shown below. Moreover, no values for $L(r)$ for reversing flows are available in the literature.

From Eq. 6, given a void distribution and liquid density, and assuming some value for the wall shear, the velocity distribution in the vessel can be determined using numerical integration; an analytic solution is not available for this system of equations. Each wall shear will yield a different velocity distribution, corresponding to a different total upward flow rate, Q, for the liquid in the column.

$$Q = \int_0^R 2\pi r U(r)\,dr \quad (=0 \text{ for zero net flow rate}) \quad (10)$$

The value for wall shear that corresponds to no net liquid flow is chosen for the final solution. Note that the user obtains the value of the wall shear from this analysis; it need not be known beforehand. Very high values of shear stress produce a distribution for which there is no change in flow direction over the column diameter, since the circulation effect is superimposed on a strong turbulent velocity profile.

Numerical Integration and Comparison with Data

Based on the equations above, a FORTRAN program was written to determine the velocity profile of the circulating liquid in a bubble column for some assumed value of wall shear stress. The value of wall shear stress was adjusted and this process

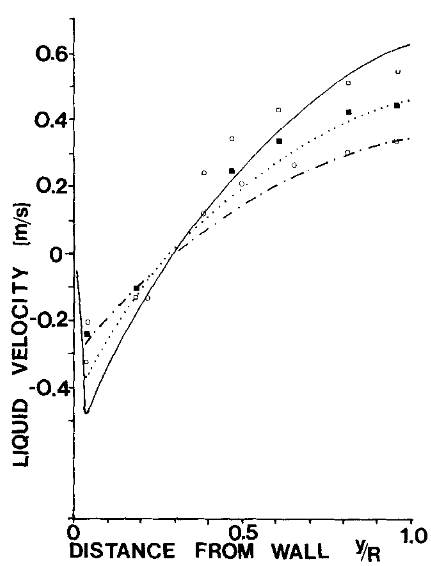

Figure 1. Comparison of predicted circulation velocity with data of Hills (1974), 138 mm column.

Pred.	Data	Gas Superficial Veloc. mm/s	Wall Shear N/m^2
———	□	169	−3.03
. . . . .	■	64	−1.83
· — · — ·	○	38	−1.15

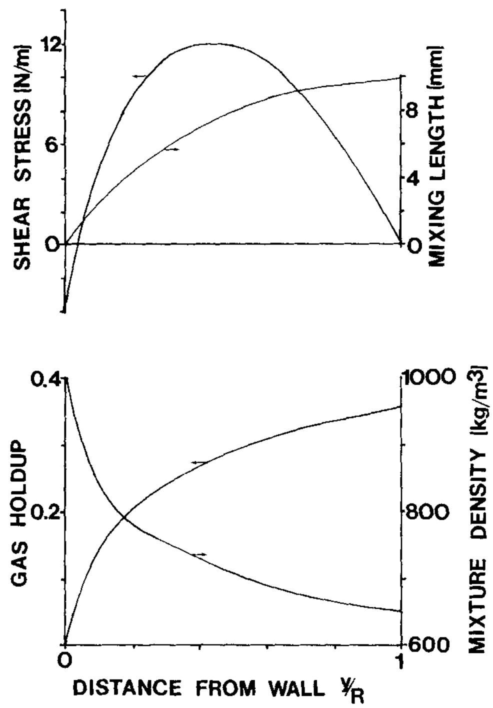

Figure 2. Profiles for four parameters corresponding to velocity distribution in Fig. 1 for a gas superficial velocity of 169 mm/s.

repeated until the overall volumetric flow across the column diameter was zero for some value of shear stress.

The new theory was compared with the data of Hills (1974) for a 138 mm dia. column. In each case the void profile supplied by Hills was fitted closely by a radial power law curve, which was in turn used in the FORTRAN program to predict the circulation velocity profile. The theoretical velocity profiles for different superficial gas velocities calculated in this manner are compared with the experimental data of Hills in Figure 1. Considering that the mixing length distributions and boundary conditions used in the program are applicable to fully developed turbulent pipe flows, agreement between theory and data is good. Deviation may also be ascribed to the shortness of the column and the measurement techniques used. It is interesting to note in Figure 1 that velocity is zero close to $r = 0.7R$; this will not necessarily be the case when there is a net flow rate of liquid in the column. Figure 2 shows the profiles of various parameters in the column as predicted by the program.

The program was also used to see whether the new model could predict another phenomenon, namely, the high wall shears encountered in some low-velocity, two-phase flows (Nakoryakov et al., 1981; Clark and Flemmer, 1985b,c; Niino et al., 1978). In low-velocity bubble flows the wall shear is often found to be far greater than in a single-phase flow at the same velocity, so that simple or homogenous pressure loss models fail dismally in this region. It is argued here that these high wall losses correspond to circulation patterns where liquid rises up the pipe wall and flows back down near the center line. These circulation patterns are induced by saddle-shaped void profiles, which have received increasing attention in the recent multiphase flow literature (Galaup, 1975; Drew and Lahey, 1981, 1982; Herringe and Davis, 1976, 1978). Such saddle-shaped profiles need not be merely an entrance effect (Clark and Flemmer, 1985a). Figure 3 shows four theoretical void profiles corresponding to a superficial gas velocity of 38 mm/s in a 138 mm column. Shear stresses, found using the FORTRAN program, corresponding to the circulation profile representing zero net liquid flow in the column are given with each profile. For the saddle-shaped profile a significant upward wall shear stress arises. In very low flow situations, this upward wall shear stress will be far greater than the shear stress that would arise in a single-phase flow with a laminar or turbulent velocity profile. In this way high shear stresses in some low-velocity bubble flows can be described by the new theory. This phenomenon will receive more detailed attention in a future paper.

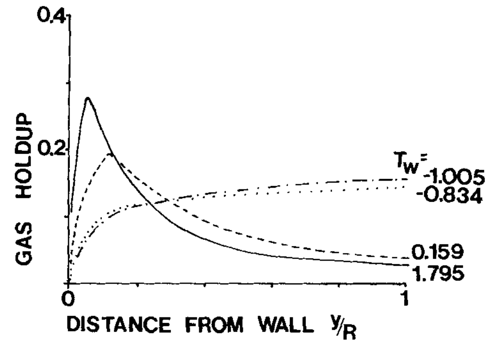

Figure 3. Effect of gas void profile on wall shear.
Each profile shows corresponding wall shear stress in N/m^2.

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Notation

g = gravitational acceleration
L = turbulent mixing length
p = power law exponent
Q = total volumetric flow rate
r, r' = radial distance from column centerline
R = column radius
T = shear stress
U = time-averaged liquid velocity
y_w = boundary layer thickness

Greek letters

ϵ = time-averaged local gas void fraction
ρ = time-averaged mixture density
ν = kinematic viscosity
μ = viscosity

Subscripts

c = at column centerline
G = gas phase
i = averaged across inner cylinder
L = liquid phase
w = at column wall

Superscripts

$\bar{}$ = averaged across column radius

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